Ferrofluid based secant shaped slider bearing

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ABSTRACT
An impermeable bearing with a secant shaped slider and a stationary stator was analysed mathematically by taking a ferrofluid lubricant whose flow followed the Jenkins flow behaviour. Expressions were obtained for various bearing characteristics. Computed values were displayed in graphical form. When the magnetization parameter was increased, load capacity of the bearing increased, coefficient of friction decreased, the position of the centre of pressure shifted towards the inlet of the bearing and the friction on the slider was unaltered. Contrary an increase in the material parameter caused decrease in load capacity, increase in friction and coefficient of friction and shift of the position of the centre of pressure towards the inlet.
Keywords : *Ferrofluid, Lubrication, Secant Shaped Slider, Bearing, Jenkins Model*

1 INTRODUCTION

Ajwalia (1984) and Cameron (1987) studied secant function shaped slider bearing with conventional lubricants because it was found that the slider did not stay flat owing to elastic, thermal and uneven wear effects. Such bearings are designed to support axial loads. In fact, Cameron analysed the secant-shaped slider bearing with a film that was initially convergent unlike we did here. He remarked that if two such shapes were put, back to back, the gap become continuous unlike in the case of two linear or exponential shapes. They are used in hydroelectric generators, stream, gas turbines and in other equipment. Bhat and Patel (1991) extended the above analysis using a magnetic fluid lubricant. They assumed that the applied magnetic field was oblique to a surface and it vanished at the inlet and outlet of the bearing as was done by Agrawal (1986). In analysis (Agrawal, 1986; Bhat and Patel, 1991) the fluid flow was governed by the Neuringer-Rosensweig model.

Patel and Deheri (2002) and Shah and Bhat (2003) studied ferrofluid lubrication of the squeeze film between two secant shaped plates and secant shaped slider bearing respectively using the Neuringer-Rosensweig model for the flow behaviour.

Recently Ram and Verma (1999) studied porous inclined slider bearing with ferrofluid lubricant flowing according to the Jenkins flow behaviour. The pressure field only is modified in the Neuringer-Rosensweig model. However, both the pressure and velocity fields are modified in the Jenkins model. The latter field is modified owing to the material constant with unit of angular momentum per unit mass of the fluid per unit field strength. However, both the models assume that the magnetization vector is parallel to the magnetic field vector. In this paper we analyse a secant shaped slider bearing with a ferrofluid lubricant whose flow is governed by the Jenkins model with the added advantage of a material constant.

2 ANALYSIS

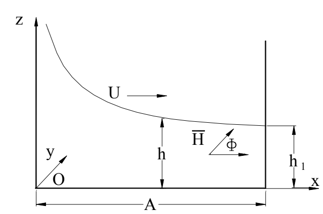

Figure 1. Secant Shaped Slider Bearing

The bearing displayed in Figure 1 consists of a secant shaped slider moving with a uniform velocity U in the x-direction and a stator lying along the x-axis. It has a length A and breadth B with A << B. The film thickness h is given by the equation

$$h = h_1 \sec\left\{\frac{\pi(A-x)}{2A}\right\}, \quad 0 < x \le A \qquad (1)$$

h_1 being the minimum film thickness.

The applied magnetic field $\overline{H}$ is inclined to the x-axis at an angle Φ which is determined as in (4,7) and it vanishes at the bearing ends so that

$H^2 = K\,x\,(A - x)$, (2)

K being a quantity introduced to suit the dimensions of both sides.

The equations of the Jenkins model for steady flow neglecting inertia terms as modified by Maugin (1980) are

$$-\nabla p + \zeta\nabla^2\overline{q} + \mu_0(\overline{M}\bullet\nabla)\overline{H} + \rho\alpha^2\nabla\times\left(\frac{\overline{M}}{M}\times\overline{M}^*\right) = 0 \tag{3}$$

$$\nabla\bullet\overline{q} = 0 \tag{4}$$

$$\nabla\times\overline{H} = 0 \tag{5}$$

$$\nabla\bullet(\overline{H}+4\pi\overline{M}) = 0 \tag{6}$$

$$\overline{M} = \overline{\mu}H \tag{7}$$

$$\text{and } \overline{M}^* = \frac{1}{2}(\nabla\times\overline{q})\times\overline{M} \tag{8}$$

where p, ζ, $\overline{q}$, μ_0, $\overline{M}$, ρ, α^2, M, $\overline{M}^*$, $\overline{\mu}$ are the film pressure, fluid viscosity, fluid velocity, free space permeability, the magnetization vector, fluid density, material constant, magnitude of $\overline{M}$, co-rotational derivative of $\overline{M}$ and magnetic susceptibility of the fluid respectively.

With the usual assumption of lubrication i.e. assuming that derivatives of velocity components across the film i.e. along z-axis predominate, we solve equation (3) for x-component of the velocity u under no-slip boundary conditions i.e. u = 0 when z = 0 and u = U when z = h, and substituting the value of u in the integral form of continuity equation, we get the Reynolds type equation as (9)

$$\frac{d}{dx}\left[\left\{\frac{h^3}{1-\frac{\rho\alpha^2\overline{\mu}H}{2\zeta}}\right\}\frac{d}{dx}\left(p-\frac{1}{2}\mu_0\overline{\mu}H^2\right)\right] = 6\zeta U\frac{dh}{dx}\ , \tag{9}$$

Using equations (1), (2) and the dimensionless quantities,

$$X = \frac{x}{A}\ ,\ ,\overline{h} = \frac{h}{h_1}\ ,\beta^2 = \frac{\rho\alpha^2\overline{\mu}\sqrt{K}A}{2\zeta}\ ,$$
$$\overline{p} = \frac{h_1^2 p}{\zeta UA}\ ,\mu^* = \frac{\mu_0\overline{\mu}KAh_1^2}{\zeta U}\ , \tag{10}$$

equation (9) can be written as

$$\frac{d}{dX}\left[C\frac{d}{dX}\left\{\overline{p}-\frac{1}{2}\mu^*X(1-X)\right\}\right] = 6\frac{d\overline{h}}{dX}\ , \tag{11}$$

where $C = \dfrac{\overline{h}^3}{(1-\beta^2\sqrt{X(1-X)})}$,

$$\overline{h} = \sec\left\{\frac{\pi}{2}(1-X)\right\}, 0 < X \le 1 \tag{12}$$

3 SOLUTIONS

Solving equation (11) under the boundary conditions

$\overline{p} = 0$ when $X = 0,1$, (13)

because $\overline{p}$ is negligible at the boundaries i.e. at inlet and outlet of the bearing.

we obtain the dimensionless pressure $\overline{p}$ as

$$\overline{p} = \frac{1}{2}\mu^*X(1-X) + \int_1^X \frac{6\overline{h}-Q}{C}dX \tag{14}$$

$$\text{where } Q = \frac{6\int_0^1 \frac{\overline{h}}{C}dX}{\int_0^1 \frac{1}{C}dX} \tag{15}$$

The dimensionless forms of load capacity W of the bearing, frictional force F on the slider, coefficient of friction f and the x-coordinate $\overline{X}$ of the centre of pressure are given by equations

$$\overline{W} = \frac{h_1^2 W}{\zeta UA^2B} = \frac{\mu^*}{12} - \int_0^1 X\frac{6\overline{h}-Q}{C}dX \tag{16}$$

$$\overline{F} = \frac{h_1 F}{\zeta UAB} = \int_0^1\left[\frac{1}{\overline{h}} + \frac{\overline{h}\left(6\overline{h}-Q\right)}{2C(1-\beta^2\sqrt{X(1-X)})}\right]dX \tag{17}$$

$$\overline{f} = \frac{Af}{h_1} = \frac{\overline{F}}{\overline{\overline{W}}} \tag{18}$$

$$\overline{Y} = \frac{\overline{X}}{A} = \frac{1}{\overline{\overline{W}}}\left[\frac{\mu^*}{24} - \frac{1}{2}\int_0^1 X^2\frac{6\overline{h}-Q}{C}dX\right] \tag{19}$$

4 DISCUSSION AND CONCLUSION

Expressions for dimensionless load capacity W, friction $\overline{F}$ on the slider, coefficient of friction $\overline{f}$ and for the position $\overline{Y}$ of the centre of pressure are given by equations (16) – (19). Their values are computed for different values of magnetization parameter μ^* and the material parameter β^2 and displayed in Figure 2-5.

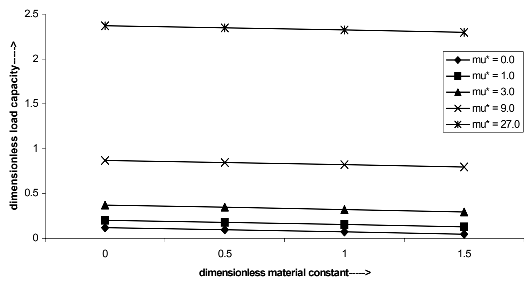

Figure 2. Dimensionless load capacity $\overline{W}$ for different values of dimensionless magnetization parameter μ^* (mu*) and dimensionless material constant β^2.

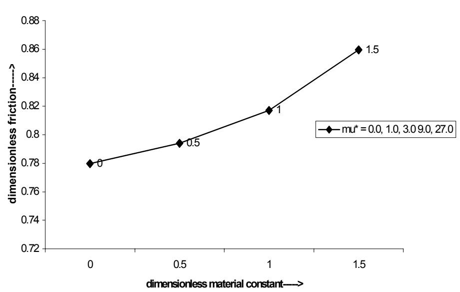

Figure 3. Dimensionless friction $\overline{F}$ for different values of dimensionless magnetization parameter μ^* (mu*) and dimensionless material constant β^2.

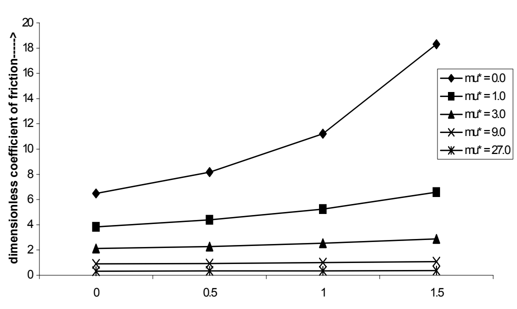

Figure 4. Dimensionless coefficient of friction $\overline{f}$ for different values of dimensionless magnetization parameter μ^* (mu*) and dimensionless material constant β^2.

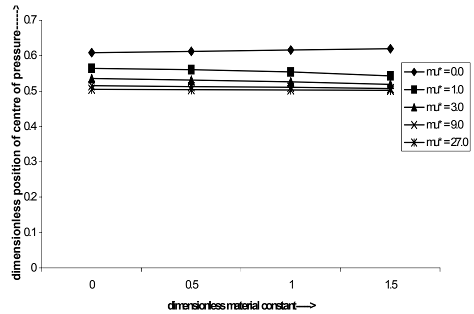

Figure 5. Dimensionless position of centre of pressure $\overline{Y}$ for different values of dimensionless magnetization parameter μ^* (mu*) and dimensionless material constant β^2.

Figure 2 shows that $\overline{W}$ increases when μ^* increases and it decreases when β^2 increases. From Figure 3, $\overline{F}$ increases with β^2 and does not depend on μ^* as seen from equation (17). It is seen from Figure 4 that $\overline{f}$ increases with β^2 and it decreases when μ^* increases. We conclude from Figure 5 that the position of the centre of pressure shifts towards the bearing inlet when μ^* or β^2 increases.

The above analysis reduces to that of Cameron (1987) if $\mu^* = 0$, $\beta^2 = 0$.

Ram and Verma (1999) obtained expressions for pressure and load capacity of a porous inclined slider bearing using the Jenkins model with some approximations. They found that the load capacity of the bearing increased with increasing magnetization parameter as we did . However, we do not agree with their result of increasing load capacity with increasing values of the material parameter. This contradictory result might be owing to their drastic approximations or unrealistic values of the parameter.

This nature of the results obtained for the various bearing characteristics is more significant than the Neuringer-Rosensweig model because of the consideration of material parameter in Jenkins model.

In the design of bearings it is better to consider rotation of the fluid, fluid mass and the field strength. The material constant may take the above three aspects in a theoretical study of the bearing. Moreover, the added advantage of secant-shaped slider from that of inclined plane, exponential and convex shaped slider is that the former is explicitly independent of the ration of the maximum film thickness to the minimum film thickness which the later are not.

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NOMENCLATURE

A	bearing length
B	bearing breadth
C	function of X
f	coefficient of friction
F	friction on the slider
h	film thickness
h_1	minimum film thickness
K	quantity with dimension A^2m^{-4}
p	film pressure
Q	constant of integration
u	fluid velocity components in the x-direction
U	velocity of slider
W	load capacity of the bearing
x,y,z	coordinates
X	x/A dimensionless coordinate
ς	fluid viscosity
μ_0	permeability of free space
$\underline{\mu}^*$	dimensionless magnetization parameter
M	magnetization vector
$\underline{M}$	magnitude of $\bar{M}$
$\underline{M}^*$	co-rotational derivative of $\bar{M}$
q	fluid velocity
ρ	fluid density
$\underline{f}$	dimensionless coefficient of friction
$\underline{F}$	dimensionless friction force on the slider
$\underline{h}$	h / h_1 , dimensionless film thickness
$\underline{H}$	applied magnetic field
$\underline{p}$	dimensionless pressure
$\underline{W}$	dimensionless load capacity
$\underline{X}$	x-coordinate of the centre of pressure
$\underline{Y}$	X / A, dimensionless $\bar{X}$
$\underline{H}$	strength of magnetic field
μ	magnetic susceptibility
Φ	inclination of $\bar{H}$ to the x- axis
α^2	material constant
β^2	dimensionless material constant