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Some Estimation Procedures for the Inverted Exponential Distribution

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Abstract

The objective of the present paper is to study the properties of different types of estimators for the parameter of an inverted exponential model. Several distributional properties of the lower record values of the model are also discussed.

1. Introduction

Exponential distribution is the most exploited distribution for the life data analysis, but its suitability is restricted to constant failure rate. For situations where the failure rate is monotonically increasing or decreasing, two-parameter Weibull and Gamma distribution are the most popular distributions used for analyzing any lifetime data. Both distributions have increasing and decreasing failure rates depending on the shape parameter. However, one of the major disadvantages of the gamma distribution is that its distribution, survival function or failure rate cannot be expressed in a closed form if the shape parameter is not an integer. This makes the gamma distribution unpopular compared to a Weibull distribution, which has a nice closed form for the failure rate and survival functions. On the other hand, the Weibull distribution has its own disadvantages. Bain & Engelhardt (1991) have pointed out that the maximum likelihood estimators of a Weibull distribution might not behave properly for all parametric ranges.

Recently two new distributions, generalized exponential and inverted exponential distribution have been introduced. The generalized exponential distribution can be used quite effectively in situations where a skewed distribution is needed. Gupta & Kundu (1999) and Raqab and Ahsanullah (2001) have investigated several properties of the two parameter generalized exponential distribution. Stefanski (1996) have discussed about some basic properties of the inverted exponential distribution. Recently, Abouammoh and Alshingiti (2009) introduced a generalized version of inverted exponential distribution and discussed the statistical and reliability properties of the distribution. Maximum likelihood estimation and least square estimation are used to evaluate the parameters and the reliability of the distribution. The inverted exponential distribution is considered as the life distribution for the present study.

It is remarkable that most of the Bayesian inference procedures have been developed with the usual squared error loss function (SELF), which is symmetrical and associates equal importance to the losses due to overestimation and underestimation of equal magnitude. However, such a restriction may be impractical in most situations of practical importance (Parsian & Kirmani, 2002). For such situations a useful asymmetric loss function was introduced by Varian (1975) called as the LINEX loss function. This function rises approximately exponentially on one side of zero and approximately linearly on the other side. A suitable alternative to the LINEX loss function is general Entropy loss function (GELF) proposed by Calabria & Pulcini (1996) and is given for the parameter θ as

$$L(\Delta) = \Delta^{C} - C\ln(\Delta) - 1 \;;\; C \neq 0,\; \Delta = \hat{\theta}\,\theta^{-1}. \tag{1.1}$$

The shape parameter C allows different shapes of this loss function. For $C > 0$, a positive error $\left(\hat{\theta} > \theta\right)$ causes more serious consequences than a negative error and vice versa.

In the present paper, the properties of the Bayes estimator, Shrinkage estimator and Minimax estimator of the parameter θ under the SELF and GELF for the inverted exponential distribution have studied. The moments of the lower record value and the estimation of the parameter, based on a series of observed record values by the maximum likelihood and moment methods are also presented.

2. The Model

The probability density function of the inverted exponential model is given as

$$f(x;\theta) = \frac{1}{\theta x^{2}} \exp\left(-\frac{1}{\theta x}\right); \quad x > 0,\; \theta > 0. \tag{2.1}$$

If $x_1, x_2, \ldots, x_n$ be the n independent random samples form the model (2.1), then the likelihood function is obtained as

$$L\left(x_1, x_2, \ldots, x_n \mid \theta\right) = \frac{1}{\theta^{n}} \left(\prod_{i=1}^{n} \frac{1}{x_i^{2}}\right) \exp\left(-\frac{T}{\theta}\right),$$

$$T = \sum_{i=1}^{n} \frac{1}{x_i}. \tag{2.2}$$

The maximum likelihood estimate (MLE) of the parameter θ is $\hat{\theta} = n^{-1}T$. The estimator $\hat{\theta}$ be the sufficient and unbiased estimator for the parameter θ. The model (2.1) has no finite moments. Further, $x_i^{-1}; i = 1, 2, \ldots, n$ are iid exponential with parameter θ, and T distributed as Gamma with probability density function

$$f(T) = \frac{T^{n-1}}{\Gamma n} e^{-T/\theta} \theta^{-n}; \; T > 0. \tag{2.3}$$

3. The Bayes Estimators and Their Properties

The natural family of conjugate prior of θ is considered as the inverted Gamma with probability density function

$$g_1(\theta) = \frac{\beta^{\alpha}}{\Gamma \alpha} \theta^{-(\alpha+1)} \exp\left(-\frac{\beta}{\theta}\right) \tag{3.1}$$

$; \theta, \alpha, \beta > 0.$

Further, in a situation where the researchers have no or very little prior information about the parameter θ, one may use a family of priors defined as

$$g_2(\theta) = \theta^{-\vartheta}; \theta, \vartheta > 0. \tag{3.2}$$

If $\vartheta = 0$ we get a diffuse prior and if $\vartheta = 1$ a non-informative prior is obtained. The posterior density of θ corresponding to the prior $g_1(\theta)$ is obtained as

$$Z_1(\theta) = \frac{(T+\beta)^{\alpha+n}}{\Gamma(\alpha+n)\theta^{\alpha+n+1}} \exp\left(-\frac{T+\beta}{\theta}\right). \tag{3.3}$$

Which is again an inverted Gamma with the parameters $(\alpha+n)$ and $(T+\beta)$. Similarly, the posterior density of θ corresponding to $g_2(\theta)$ is given by

$$Z_2(\theta) = \frac{T^{n+\vartheta-1}}{\Gamma(n+\vartheta-1)\theta^{n+\vartheta}} \exp\left(-\frac{T}{\theta}\right). \tag{3.4}$$

The Bayes estimator of θ under SELF-criterion, corresponding to $Z_1(\theta)$ is

$$\hat{\theta}_{B1} = E_P(\theta) = \frac{T+\beta}{n+\alpha-1}. \tag{3.5}$$

Here, the suffix P indicates that the expectation taken under the posterior density $Z_1(\theta)$. Similarly, the Bayes estimator of θ under the GELF is obtained with respect to $Z_1(\theta)$ as

$$\hat{\theta}_{B2} = \left(E_P\left(\theta^{-C}\right)\right)^{-1/C} = \left(\frac{\Gamma(n+\alpha)}{\Gamma(n+\alpha+C)}\right)^{1/C}(T+\beta). \tag{3.6}$$

The expressions of the risks under the SELF and GELF are given respectively as

$$R_S\left(\hat{\theta}_{Bi}\right) = n\lambda_i^2\theta^2 + \left(\lambda_i\beta + \theta(n\lambda_i - 1)\right)^2 \tag{3.7}$$

and

$$R_G\left(\hat{\theta}_{Bi}\right) = J\left(\Delta_i^C\right) - CJ\left(\log\Delta_i\right) - 1; \tag{3.8}$$

where

$$\lambda_1 = \frac{1}{n+\alpha-1}, \; \lambda_2 = \left(\frac{\Gamma(n+\alpha)}{\Gamma(n+\alpha+C)}\right)^{1/C},$$

$$\Delta_i = \lambda_i\left(\omega + \frac{\beta}{\theta}\right), \; J(\underline{\omega}) = \frac{1}{\Gamma n}$$

$\int_0^{\infty} \underline{\omega}\, e^{-\omega}\omega^{n-1} d\omega$, $i = 1, 2$ and $\underline{\omega}$ be the function of ω. The suffixes S and G respectively indicates the loss criterion SELF and GELF.

The relative efficiency between $\hat{\theta}_{B1}$ and $\hat{\theta}_{B2}$ under both risks criterion are defined as

$$RE_S\left(\hat{\theta}_{B2}, \hat{\theta}_{B1}\right) = \frac{R_S\left(\hat{\theta}_{B1}\right)}{R_S\left(\hat{\theta}_{B2}\right)}$$ and

$$RE_G\left(\hat{\theta}_{B2}, \hat{\theta}_{B1}\right) = \frac{R_G\left(\hat{\theta}_{B1}\right)}{R_G\left(\hat{\theta}_{B2}\right)}.$$

Remark:

All the results discussed above holds for the posterior distribution $Z_2(\theta)$ if following substitution $\alpha(=\vartheta-1)$ and $\beta(=0)$ have been made.

4. The Minimax Estimators and Their Properties

The basic principle of this approach is to minimize the loss. The derivation depends primarily on a theorem, which is due to Hodge & Lehmann (1950) and can be stated as follows

Let $\tau = \left\{F_\theta : \theta \in \Theta\right\}$ be a family of distribution function and D be a class of estimator of the parameter θ. Suppose that $d^* \in D$ is a Bayes estimator against a prior distribution $\pi(\theta)$ on the parameter space Θ. Then the Bayes estimator d^* is said to be minimax estimator if the risk function of d^* is independent on Θ.

It is clear that the equation (3.7) and (3.8) involve the parameter θ. Hence, the Bayes estimators $\hat{\theta}_{B1}$ and $\hat{\theta}_{B2}$ are not a minimax estimator.

Now, the Bayes estimators corresponding to the posterior $Z_2(\theta)$ are obtained respectively under both loss criterion as

$$\hat{\theta}_{B3} = \frac{T}{n+\vartheta-2} \text{ and}$$

$$\hat{\theta}_{B4} = \left(\frac{\Gamma(n+\vartheta-1)}{\Gamma(n+\vartheta-1+C)}\right)^{1/C} T. \quad (4.1)$$

The risks of these Bayes estimators corresponding to the SELF and GELF are given respectively as

$$R_S(\hat{\theta}_{B3}) = \theta^2\left\{n(n+1)\lambda_3^2 + 1 - 2n\lambda_3\right\};$$

$$\lambda_3 = \frac{1}{n+\vartheta-2} \quad \text{and}$$

$$R_G(\hat{\theta}_{B4}) = J(\lambda_4^C) - CJ(\log\lambda_4) - 1$$

$$;\ \lambda_4 = \omega\left(\frac{\Gamma(n+\vartheta-1)}{\Gamma(n+\vartheta-1+C)}\right)^{1/C}.$$

It is observed that the Bayes estimator $\hat{\theta}_{B3}$ not a minimax estimator. But the risk of Bayes estimator $\hat{\theta}_{B4}$ is independent of the parameter θ. Hence, $\hat{\theta}_{B4}$ is a minimax estimator under the GELF criterion.

Now, the invariant form of the SELF (ISELF) is defined as

$$L_{IS}(\hat{\theta}) = \left(\frac{\hat{\theta}-\theta}{\theta}\right)^2. \quad (4.2)$$

Corresponding to the posterior $Z_2(\theta)$, the Bayes estimator of θ under ISELF is obtained as

$$\hat{\theta}_{B5} = \frac{E_P(\theta^{-1})}{E_P(\theta^{-2})} = \frac{T}{n+\vartheta} \quad (4.3)$$

with the risk

$$R_{IS}(\hat{\theta}_{B5}) = n(n+1)\lambda_5^2 + 1 - 2n\lambda_5;\ \lambda_5 = \frac{1}{n+\vartheta}. \quad (4.4)$$

Hence, the Bayes estimator $\hat{\theta}_{B5}$ is also the minimax estimator under the ISELF.

The following statistical problem is equivalent to some two person zero sum game between the Statistician (Player - II) and Nature (Player - I). Here the pure strategies of Nature are the different values of θ in the interval $(0,\infty)$ and the mixed strategies of Nature are the prior densities of θ in the interval $(0,\infty)$. The pure strategies of Statistician are all possible decision functions in the interval $(0,\infty)$.

The expected value of the loss function is the risk function and it is the gain of the Player - I. Further, the Bayes risk is defined as

$$R^*(\eta, \hat{\theta}_B) = E_\theta R(\hat{\theta}_B).$$

Here, the expectation has been taken under the prior density of parameter θ. If the loss function is continuous in both the estimator $\hat{\theta}_B$ and parameter θ, and convex in $\hat{\theta}_B$ for each value of θ then there exist measures η^* and $\hat{\theta}_B^*$ for all θ and $\hat{\theta}_B$ so that, the relation holds

$$R^*(\eta, \hat{\theta}_B^*) \le R^*(\eta^*, \hat{\theta}_B^*) \le R^*(\eta^*, \hat{\theta}_B).$$

The number $R^*(\eta^*, \hat{\theta}_B^*)$ is known as the value of the game, and η^* and $\hat{\theta}_B^*$ are the corresponding optimum strategies of the Player I and II. In statistical terms η^* is the least favorable prior density of θ and the estimator $\hat{\theta}_B^*$ is the minimax estimator. In fact, the value of the game is the loss of the Player – II. Hence, the optimum strategy of Player – II and the value of game are given as

Optimum Strategy	**Corresponding Loss**	**Value of Game**
$\hat{\theta}_{B4}$	GELF	$J(\lambda_4^C) - CJ(\log\lambda_4) - 1$
$\hat{\theta}_{B5}$	ISELF	$n(n+1)\lambda_5^2 + 1 - 2n\lambda_5$

5. The Shrinkage Estimators and Their Properties

It is recognized that a shrinkage estimator performs better if a guess value of the parameter is in the vicinity of the true value and the sample size is small. Following Thompson (1968), a shrinkage estimator of θ is defined as

$$\hat{\theta}_{SH} = k(\hat{\theta} - \theta_0) + \theta_0, \quad (5.1)$$

where θ_0 is a guess value of the parameter θ. The shrinkage factor k lies between zero and one, and is specified by the experimenter according to his belief in the guess value θ_0. Several authors have studied the

performances of the shrinkage estimators utilizing a point guess value. Few resents works related to the shrinkage are Pandey & Singh (2002), Shirke (2004), Singh and Saxena (2005), Prakash & Singh (2006), Singh et al. (2007), Prakash et al. (2008) and others in different context.

The risk under the SELF and the GELF for the shrinkage estimator $\hat{\theta}_{SH}$ are obtained respectively as

$$R_S\left(\hat{\theta}_{SH}\right) = \theta^2\left\{k^2\left(\frac{n+1}{n}+\delta\,(\delta-2)\right)+(1-\delta)^2(1-2k)\right\}$$

$$;\ \delta = \frac{\theta_0}{\theta} \tag{5.2}$$

and

$$R_G\left(\hat{\theta}_{SH}\right) = J\left(\bar{\Delta}^C\right) - C\,J\left(\log\bar{\Delta}\right) - 1\ ;$$

$$\bar{\Delta} = k\left(\frac{\omega}{n}-\delta\right)+\delta\,. \tag{5.3}$$

The value of the shrinkage factor k that minimizes the risk under the SELF (5.2) is given as

$$k_{min} = (1-\delta)^2\left(\frac{n+1}{n}+\delta\,(\delta-2)\right)^{-1}. \tag{5.4}$$

The unknown parameter θ involved in k_{min}. Replacing δ by its estimate $\hat{\delta} = \theta_0 / \hat{\theta}$ in (5.4), an estimate of $k = k_1$ (say) is obtained.

Similarly, the shrinkage factor $k = k_2$ (say) which minimizes the risk (5.3) of the estimator $\hat{\theta}_{SH}$ under GELF is obtained by solving the given equation

$$J\left(\left(\frac{\omega}{n}-\delta\right)\bar{\Delta}^C - 1\right) = J\left(\left(\frac{\omega}{n}-\delta\right)\bar{\Delta} - 1\right). \tag{5.5}$$

Therefore, the improved shrinkage estimators are given as

$$\hat{\theta}_{SH\,i} = k_i\left(\hat{\theta} - \theta_0\right) + \theta_0\ ;\ i = 1, 2. \tag{5.6}$$

The relative bias of the shrinkage estimators $\hat{\theta}_{SH1}$ and $\hat{\theta}_{SH2}$ are obtained as

$$RB\left(\hat{\theta}_{SH\,i}\right) = \frac{E\left(\hat{\theta}_{SH\,i}\right)}{\theta} - 1 = J\left(\Delta_i\right) - 1\ ;$$

$$\text{where } \Delta_i = k_i\left(\frac{\omega}{n}-\delta\right) + \delta \text{ and } i = 1, 2. \tag{5.7}$$

Also, the expression of the risks under both loss criterion are

$$R_S\left(\hat{\theta}_{SH\,i}\right) = J\left(\Delta_i^2 - 2\Delta_i + 1\right)$$

$$\text{and} \tag{5.8}$$

$$R_G\left(\hat{\theta}_{SH\,i}\right) = J\left(\Delta_i^C\right) - C\,J\left(\log\Delta_i\right) - 1\ ;\ i = 1, 2. \tag{5.9}$$

The relative efficiency between the shrinkage estimators $\hat{\theta}_{SH\,1}$ and $\hat{\theta}_{SH\,2}$ are defined as

$$RE_S\left(\hat{\theta}_{SH\,2}, \hat{\theta}_{SH\,1}\right) = \frac{R_S\left(\hat{\theta}_{SH\,1}\right)}{R_S\left(\hat{\theta}_{SH\,2}\right)}$$

and

$$RE_G\left(\hat{\theta}_{SH\,2}, \hat{\theta}_{SH\,1}\right) = \frac{R_G\left(\hat{\theta}_{SH\,1}\right)}{R_G\left(\hat{\theta}_{SH\,2}\right)}.$$

6. Distributional Properties of Lower Record Values

There are several situations in which we are required to keep the knowledge of maximum (minimum) observations available in the data and to discard all other observations which are smaller (larger) than the observations already chosen. The specific observation chosen is generally referred to as a record. The branch of statistics that helps us in study of statistical properties of these records is called the Record statistics. The concept of record values and record statistics was first introduced by Chandler (1952) and developed by Feller (1966) in connection with gambling problem. Record values appear in many statistical applications. Record values are closely connected with the occurrence times of some corresponding non - homogeneous Poisson process used in so - called Shock models. It may be helpful as a model for successively largest values for highest water levels or highest temperatures.

In the context of bioscience, like as the behavior of human organs as in kidneys or lungs. Similarly, in the assessment of glucose level among diabetic patients, the researcher may be interested to study the behavior of the ordered records of glucagons. A lot of works has been done in the field of the record value. See details for Nagaraja (1988), Ahsanullah (1995, 2006), and Raqab (2002).

Let us assume a sequence of random variables $\{x_1, x_2, x_3, \ldots\}$ from the considered model (2.1). Let $Y_m = \max(\min)\{x_1, x_2, \ldots, x_m : m \geq 1\}$, then Y_j is an upper (lower) record value

of $\left\{ x_m : m \geq 1 \right\}$ if $Y_j > Y_{j-1}$ or $Y_j < Y_{j-1}$ for $j > 1$.

It is evident from the above definition of the record value that Y_1 is an upper as well as lower record value.

Following Ahsanullah (1992), the marginal and joint probability density function for the rth and s^{th} lower records are given as

$$f_r\left(x_{L(r)}\right) = \frac{1}{\Gamma(r)} f\left(x_r\right) \left(H\left(x_r\right)\right)^{r-1}$$

$$; H\left(x_r\right) = -\log F\left(x_r\right) \tag{6.1}$$

and

$$f_{r,s}\left(x_{L(r)}, x_{L(s)}\right)$$

$$= \frac{1}{\Gamma(r)\,\Gamma(s-r)} f\left(x_s\right)\left(H\left(x_r\right)\right)^{r-1}\left(-H'\left(x_r\right)\right)$$

$$\left(H\left(x_s\right) - H\left(x_r\right)\right)^{s-r-1};$$

$$H'\left(x_r\right) = \frac{\partial}{\partial x_r} H\left(x_r\right), 0 \leq x_s \leq x_r \leq \infty . \tag{6.2}$$

Using the relations (6.1) & (2.1), the distribution of mth lower record value of the model (2.1) is obtained as

$$f_m\left(x_{L(m)}\right) = \frac{1}{\Gamma(m)} \exp\left(-\frac{1}{x\theta}\right) \frac{\theta^{-m}}{x^{m+1}} . \tag{6.3}$$

Similarly, the joint probability density function of the rth and s^{th} lower record values is obtained as

$$f_{r,s}\left(x_{L(r)}, x_{L(s)}\right) =$$

$$\frac{\theta^{-s}}{\Gamma(r)\,\Gamma(s-r)} \exp\left(-\frac{1}{y\theta}\right) \frac{(x-y)^{s-r-1}}{x^s y^{s-r+1}} . \tag{6.4}$$

6.1 The kth Moments

The kth moments of the distribution for the lower record value is given as

$$E\left(x^k_{L(m)}\right) = \frac{\Gamma(m-k)}{\Gamma(m)} \theta^{-k} . \tag{6.5}$$

The mean and variance of the lower record value is

$$\text{Mean}\left(x_{L(m)}\right) = \frac{\theta^{-1}}{m-1} \quad \text{and}$$

$$\text{Var}\left(x_{L(m)}\right) = \frac{\theta^{-2}}{(m-1)^2 (m-2)} .$$

It is observed that as the parameter θ increases, $\text{Var}\left(x_{L(m)}\right)$ decreases and as $m \to \infty$, $\text{Var}\left(x_{L(m)}\right) \to 0$.

6.2 The Joint Moment for Lower Record Values

The joint moment for the lower record values is obtained as

$$E\left(x_{L(r)}, x_{L(s)}\right) =$$

$$\int_{y=0}^{\infty} \int_{x=y}^{\infty} x\,y\, f_{r,s}\left(x_{L(r)}, x_{L(s)}\right) dx\, dy$$

$$= \frac{\theta^{-2}}{(r-1)(s-2)} . \tag{6.6}$$

Therefore, the covariance and the correlation coefficient between r^{th} and s^{th} lower record values are given by

$$\text{Cov}\left(x_{L(r)}, x_{L(s)}\right) =$$

$$\frac{\theta^{-2}}{(r-1)(s-1)(s-2)}$$

and

$$\rho\left(x_{L(r)}, x_{L(s)}\right) = \sqrt{\frac{r-2}{s-2}} .$$

6.3 Method of Maximum Likelihood

The logarithm of the likelihood function based on the lower record value is

$$\log L = \sum_{i=1}^{m-1} \log\left(\frac{f(x_i)}{F(x_i)}\right) + \log f_m(x_m) \tag{6.7}$$

Therefore, the estimate of the parameter θ based on the method of maximum likelihood is given as

$$\hat{\theta}_{ML} = \left(x_m (2m-1)\right)^{-1} .$$

6.4 Method of Moments

Following Ahsanullah (1992),

$$E\left(\bar{X}\right) = E\left(\left(x_{L(1)} + x_{L(2)} + \ldots + x_{L(m)}\right) \Big/ m\right)$$

$$= \frac{1}{m\theta} \sum_{i=1}^{m} \frac{\Gamma(i-1)}{\Gamma(i)} . \tag{6.8}$$

Thus, the estimate of the parameter θ is

$$\hat{\theta}_M = \frac{1}{m\,\bar{x}} \sum_{i=1}^{m} \frac{\Gamma(i-1)}{\Gamma(i)} .$$

Table 1

Relative efficiency between the Bayes Estimators $\hat{\theta}_{B1}$ and $\hat{\theta}_{B2}$

		$RE_S\left(\hat{\theta}_{B2}, \hat{\theta}_{B1}\right)$			
		C			
n	β, α	- 2.00	- 1.00	1.00	2.00
05	02, 03	1.5003	1.4393	1.2115	1.1022

	10, 06	1.7638	1.5586	1.2300	1.1068
	30, 11	2.0465	1.7474	1.2966	1.1349
10	02, 03	1.3649	1.3344	1.1806	1.0906
	10, 06	1.5739	1.4493	1.2043	1.0976
	30, 11	1.6996	1.5385	1.2391	1.1134
15	02, 03	1.2845	1.2646	1.1509	1.0775
	10, 06	1.4586	1.3709	1.1789	1.0876
	30, 11	1.5470	1.4517	1.1994	1.1075

$RE_G\left(\hat{\theta}_{B2}, \hat{\theta}_{B1}\right)$

05	02, 03	2.4759	1.8945	1.2816	1.1203
	10, 06	2.5245	1.9618	1.2857	1.1214
	30, 11	2.6570	2.0267	1.3393	1.1452
10	02, 03	2.0843	1.6970	1.2450	1.1062
	10, 06	2.1345	1.7227	1.2456	1.1078
	30, 11	2.1574	1.7525	1.2743	1.1217
15	02, 03	1.8649	1.5777	1.2078	1.0911
	10, 06	1.8789	1.5798	1.2159	1.0967
	30, 11	1.9296	1.6582	1.2346	1.1090

7. Numerical Analysis

7.1 The Bayes Estimators and Their Properties

The expressions of the relative efficiency involve n, θ, C and the prior parameters α and β. The values of α and β are chosen so as to keep the prior variance to be 1.00 with the considered values $(\beta, \alpha) = (02, 03), (10, 06)$ and $(30, 11)$. Other considered values are $n = 05, 10, 15$; $\theta = 04$ and $C = \pm 1, \pm 2$. The relative efficiency have been calculated and presented in the Table 01.

It is observed that the Bayes estimator $\hat{\theta}_{B2}$ performs uniformly well with respect to Bayes estimator $\hat{\theta}_{B1}$ under both loss criterion. The relative efficiency decreases as the sample size n increases when other parametric values are fixed. Opposite trend has been seen when combination of the prior parameter increase. Further, it is also noted that for C, the negative values produces higher efficiency corresponding to positive value.

7.2 The Minimax Estimators and Their Properties

The expressions of the risk for the minimax estimators involve n, ϑ and C. For the similar set of values as considered earlier with $\vartheta = 1.00, 1.50, 05, 10$ the estimated risk has been calculated and presented them in Table 02.

It is seen that the estimated risk increases (decreases) as $\vartheta(n)$ increases for both minimax estimator $\hat{\theta}_{B4}$ and $\hat{\theta}_{B5}$ under their corresponding risk criterion. Further, it is also noted that the negative values of C produces higher risks for the estimator $\hat{\theta}_{B4}$ corresponding to positive values of C.

Table 2

Estimated risk of the Minimax estimators θ_{B4} and θ_{B5}

		$R_{IS}\left(\theta_{B5}\right)$			
		n			
n ↓	C ↓	1.00	2.50	5.00	10.00
05		0.1667	0.2000	0.3000	0.4667
10		0.0909	0.1040	0.1556	0.2750
15		0.0625	0.0694	0.1000	0.1840
$R_G\left(\theta_{B4}\right)$					
05	– 2.00	0.5273	0.8659	2.6536	9.9624
	- 1.00	0.1198	0.1764	0.4267	1.1912
	1.00	0.1033	0.1349	0.2467	0.4901
	2.00	0.3890	0.4898	0.8209	1.4777
10	– 2.00	0.2268	0.2863	0.6203	2.0299
	- 1.00	0.0545	0.0670	0.1312	0.3614
	1.00	0.0508	0.0602	0.1016	0.2190
	2.00	0.1970	0.2298	0.3674	0.7261
15	– 2.00	0.1447	0.1686	0.3064	0.9003
	- 1.00	0.0353	0.0406	0.0697	0.1811
	1.00	0.0337	0.0381	0.0596	0.1278
	2.00	0.1319	0.1480	0.2231	0.4436

7.3 The Shrinkage Estimators and their Properties

The expressions of the relative biases and the relative efficiencies for the shrinkage estimators $\hat{\theta}_{SH\,1}$

Table 3

Relative Biases for the Shrinkage estimators $\hat{\theta}_{SH\,1}$ and $\hat{\theta}_{SH\,2}$

		RB $\left(\hat{\theta}_{SH\,1}\right)$						
		δ						
n	C	0.25	0.50	0.75	1.00	1.25	1.50	1.75
05		-0.0191	-0.0154	-0.0102	-0.0042	0.0048	0.0074	0.0097
10		-0.0164	-0.0107	-0.0086	-0.0018	0.0041	0.0072	0.0082
15		-0.0156	-0.0082	-0.0073	-0.0018	0.0027	0.0065	0.0067
RB $\left(\hat{\theta}_{SH\,2}\right)$								
05	-2.00	-0.1103	-0.0750	-0.0103	0	0.0375	0.0475	0.0715
	-1.00	-0.0608	-0.0116	-0.0093	0	0.0202	0.0305	0.0317
	1.00	-0.0808	-0.0120	-0.0105	0	0.0312	0.0315	0.0407
	2.00	-0.0768	-0.0115	-0.0098	0	0.0155	0.0165	0.0177
10	-2.00	-0.1070	-0.0728	-0.0100	0	0.0364	0.0461	0.0694
	-1.00	-0.0590	-0.0113	-0.0090	0	0.0196	0.0296	0.0307
	1.00	-0.0784	-0.0116	-0.0102	0	0.0303	0.0306	0.0395
	2.00	-0.0745	-0.0112	-0.0095	0	0.0150	0.0160	0.0172
15	-2.00	-0.1031	-0.0701	-0.0096	0	0.0351	0.0444	0.0669
	-1.00	-0.0568	-0.0108	-0.0087	0	0.0189	0.0285	0.0296
	1.00	-0.0755	-0.0112	-0.0098	0	0.0292	0.0295	0.0381
	2.00	-0.0718	-0.0108	-0.0092	0	0.0145	0.0154	0.0165

and $\hat{\theta}_{SH\,2}$ are the functions of n, C and δ. The numerical finding presented in the Table 03 & 04 respectively for the similar set of values as considered earlier with $\delta = 0.25\ (\ 0.25\ )\ 1.75$.

7.3.1 Under the SELF

It is observed that the relative biases negative for $\delta \le 1.00$ and positive otherwise (Table 03). As the sample size n increases the absolute relative bias (ARB) decreases. The ARB first decreases up to zero and then increases as δ increases.

The shrinkage estimator $\hat{\theta}_{SH\,2}$ performs well with respect to the shrinkage estimator $\hat{\theta}_{SH\,1}$ in the interval $0.50 \le \delta \le 1.50$ and the efficiencies decreases as n increases (Table 04). In addition, the relative efficiency increases as $|C|$ increases.

7.3.2 Under the GELF

It has been seen that the relative biases negative for $\delta < 1.00$ and zero for $\delta = 1.00$ and positive otherwise (Table 03). Other properties are similar to $\hat{\theta}_{SH\,1}$. The ARB decreases when positive C increases. Opposite trend has been seen for negative values of the C.

The shrinkage estimator $\hat{\theta}_{SH\,2}$ performs well in the effective interval $0.50 \le \delta \le 1.50$ with respect to $\hat{\theta}_{SH\,1}$ under the GELF. Other properties are similar to $\hat{\theta}_{SH\,1}$.

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Table 4

Relative Efficiency between the Shrinkage estimators $\hat{\theta}_{SH\,1}$ and $\hat{\theta}_{SH\,2}$

		$RE_S\left(\hat{\theta}_{SH\,2}, \hat{\theta}_{SH\,1}\right)$						
		δ						
n	C	0.25	0.50	0.75	1.00	1.25	1.50	1.75
05	-2.00	1.0866	1.5022	2.9942	3.6534	2.7587	2.0799	1.3761
	-1.00	0.8730	1.1409	2.9194	3.6134	2.6613	1.8359	0.9333
	1.00	0.8631	1.1309	1.7395	3.0092	2.3494	1.7359	0.8933
	2.00	0.7653	1.2208	1.8216	3.6514	2.6613	1.8359	0.9133
10	-2.00	1.0688	1.4776	2.9451	3.5935	2.7135	2.0458	1.3535
	-1.00	0.8587	1.1222	2.8715	3.5541	2.6177	1.8058	0.9180
	1.00	0.8489	1.1124	1.7110	2.9598	2.3109	1.7074	0.8786
	2.00	0.7527	1.2008	1.7917	3.5915	2.6177	1.8058	0.8983
15	-2.00	1.0406	1.4386	2.8673	3.4986	2.6419	1.9918	1.3178
	-1.00	0.8360	1.0926	2.7957	3.4603	2.5486	1.7581	0.8938
	1.00	0.8265	1.0830	1.6658	2.8817	2.2499	1.6623	0.8554
	2.00	0.7328	1.1691	1.7444	3.4967	2.5486	1.7581	0.8746
$RE_G\left(\hat{\theta}_{SH\,2}, \hat{\theta}_{SH\,1}\right)$								
n	C	0.25	0.50	0.75	1.00	1.25	1.50	1.75
5	-2.00	0.1686	1.1355	1.2135	4.9949	2.3182	1.6120	1.1842
	-1.00	1.5801	1.1262	1.1863	3.9143	2.0775	1.2642	0.9382
	1.00	1.2501	1.2186	2.1174	4.6907	3.0612	2.1452	1.0209
	2.00	1.2012	1.2905	3.0542	5.7099	3.9801	2.2744	1.0458
10	-2.00	0.1658	1.1169	1.1936	4.9130	2.2802	1.5856	1.1648
	-1.00	1.5542	1.1077	1.1668	3.8501	2.0434	1.2435	0.9228
	1.00	1.2296	1.1986	2.0827	4.6138	3.0110	2.1100	1.0042
	2.00	1.1815	1.2693	3.0041	5.6163	3.9148	2.2371	1.0286
15	-2.00	0.1614	1.0874	1.1621	4.7833	2.2200	1.5437	1.1340
	-1.00	1.5132	1.0785	1.1360	3.7485	1.9895	1.2107	0.8984
	1.00	1.1971	1.1670	2.0277	4.4680	3.0014	2.0178	0.9777
	2.00	1.1503	1.2358	2.9248	4.1492	2.9315	2.0043	1.0014